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This project is focusing on problems in operator theory and matrix theory that underlie the maximum entropy principle in signal processing and system theory. We have found generalizations of this principle for finite-dimensional problems to certain broad classes of hermitian band matrices, including Toeplitz matrices, and we have shown why a similar principle cannot exist for all hermitian band matrices.

Zeros of orthogonal polynomials are studied in a setting that generalizes the usual minimum phase theorem for the error prediction filters related to stationary time series. In another paper, we analyze the number of negative eigenvalues of an extension of a hermitian band matrix in terms of the entries in the band matrix. From this we obtain results on maximum entropy and on singular values of extensions of triangular matrices. The latter results are related to the finite-dimensional model reduction problem for linear systems.

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1988 ANNUAL REPORT

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APPLICATIONS OF OPERATOR THEORY TO MAXIMUM ENTROPY PROBLEMS

REPORT ON COMPLETED WORK

This project is concerned with problems in operator theory and matrix theory that underlie the maximum entropy principle in signal processing and system theory. We have found generalizations of this principle for finite-dimensional problems to certain broad classes of selfadjoint band matrices, and we have examined some cases in which a maximum entropy principle exists only as a constrained maximum. Our work in [2] makes it clear why a maximum entropy principle cannot exist in all cases: in general the band extension of a selfadjoint matrix is only a stationary point for the determinant function, and can yield a maximum, a minimum, or a saddle point.

For Toeplitz matrices, however, the indefinite selfadjoint case is close to the positive definite case that arises in signal processing and other situations involving stationary stochastic processes. In order to state one such result, we need some definitions. An $n \times n$ matrix R is called an *m-band matrix*, where $1 \le m \le n-2$, if the entries r_{ij} in R satisfy $r_{ij} = 0$ for $|i-j| > m$. An *extension* of R is an $n \times n$ matrix whose entries agree with those of R inside the band $|i-j| \le m$. A *band extension* of R is an invertible extension F of R such that F^{-1} is again an m-band matrix. Finally, we let $R(j,\dots,k)$ denote the submatrix of R consisting of the entries $r_{\alpha\beta}$ such that $j \le \alpha, \beta \le k$. The result is as follows:

Suppose that R is a selfadjoint Toeplitz m-band matrix such that $\det R(1,\dots,m) > 0$ and $\det R(1,\dots,m+1) > 0$. Then R admits a unique band extension F, the matrix F is Toeplitz, and F is characterized by a maximum entropy principle for matrices in the sense that, if G is any selfadjoint (but not necessarily Toeplitz) extension of R such that $\det G(1,\dots,k) > 0$ for $k = m+2,\dots,n$, then

$$\det G \le \det F,$$

with equality only if $G = F$.

A continuation of our work in [2] appears in [3]. Here we analyze the number of negative eigenvalues of an extension of a selfadjoint band matrix in terms of the entries in the band matrix and the nature of the extension. Applications are given to Toeplitz matrices and to maximum entropy. We also study the problem of determining the size of the singular values of an extension of a lower-triangular matrix. As we mentioned in our proposal, this has connections with the finite-dimensional model reduction problem. Our results in [2] and [3] reveal certain difficulties associated with infinite-dimensional generalizations, and we plan to explore these difficulties in the near future.

The results in [2] and [3] for finite Toeplitz matrices and our plans for infinite-dimensional generalizations led to the problem of the distribution of the zeros of polynomials that are orthogonal on the unit circle with respect to an indefinite inner product. This topic is connected to signal processing in the following way. Let f be the power spectrum of a single channel stationary complex time series. Then f is a positive real-valued function on $[-\pi,\pi]$. Denote the jth Fourier coefficient of f by c_j. Define an inner product on the space of complex polynomials by

$$\langle p,q\rangle = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)p(e^{i\theta})\overline{q(e^{i\theta})}\,d\theta \qquad (1)$$

for polynomials p and q. Equivalently, if n is the larger of the degrees of p and q, and if

$$\begin{aligned} p(z) &= p_0z^n + p_1z^{n-1} + \dots + p_n \\ q(z) &= q_0z^n + q_1z^{n-1} + \dots + q_n \end{aligned}$$

then

$$\langle p,q\rangle = [\bar{q}_0 \;\dots\; \bar{q}_n]\begin{bmatrix} c_0 & c_{-1} & \cdots & c_{-n} \\ c_1 & c_0 & & \\ \vdots & & & \vdots \\ c_n & & \cdots & c_0 \end{bmatrix}\begin{bmatrix} p_0 \\ p_1 \\ \vdots \\ p_n \end{bmatrix}$$

Suppose that complex numbers $x_0,\dots,x_n$, with $x_0 \neq 0$, satisfy

$$\begin{bmatrix} c_0 & c_{-1} & \cdots & c_{-n} \\ c_1 & c_0 & & \vdots \\ \vdots & & & \\ c_n & & \cdots & c_0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{2}$$

and let $f_n(z) = x_0 z^n + \dots + x_n$. Then f_n is called *the nth orthogonal polynomial on the unit circle with respect to the inner product* (1). An important property of f_n is that its zeros all lie inside the unit circle in $\mathbb{C}$. In signal processing, f_n (up to a scalar multiple) is sometimes called the nth order backward error prediction filter, and a variant of (2) is refered to as the Yule-Walker equations. The uses of such orthogonal polynomials in signal processing are described in the survey article [6].

Suppose that the function f in (1) is nonzero but not necessarily positive. In this case (1) determines an indefinite inner product. Orthogonal polynomials may be defined as before. In [7], M.G. Krein stated the following remarkable characterization of orthogonal polynomials: A polynomial

$$f_n(z) = x_0 z^n + x_1 z^{n-1} + \dots + x_n,$$

with $x_0 \neq 0$, is the n-th orthogonal polynomial on the unit circle for some indefinite inner product (1) if and only if x_0 is real and f_n has no zeros on the unit circle and no pair of zeros that are symmetric with respect to the unit circle. We have supplied in [4] a new proof of this result that contains an explicit formula for the Fourier coefficients of the weight function. In addition, we have provided a transparent proof of another theorem in [7] that describes the number of zeros of f_n that lie inside the unit circle. This theorem generalizes the standard minimum phase theorem for error prediction filters. Some generalizations of Krein's results are also presented. Our proofs make essential use of the properties of one-step extensions of self-adjoint Toeplitz matrices we developed earlier in [2]. Our work in [4] has

led to generalizations of the theory to block matrix orthogonal polynomials. See the paper in [5].

Related work on interpolation problems for rational matrix functions will appear in [1]. In this paper some basic interpolation problems are treated from a unified point of view. Included is the problem of reconstructing a rational matrix function from its null and pole data, and the problem of constructing a unitary rational matrix function when only a part of the null and pole data is given. Problems of Nevanlina-Pick type are also considered The solutions of these problems are given in an explicit form, within the framework of realization theory and finite-dimensional matrix analysis.

WORK IN PROGRESS

A limiting case of maximum entropy spectra of stationary time series occurs when the power spectrum consists of a set of pure line spectra and the time function is a sum of pure frequencies. In this case the autocorrelation matrix T_n is singular for $n \geq N$, and (2) becomes

$$\begin{bmatrix} c_0 & c_{-1} & \cdots & c_{-n} \\ c_1 & c_0 & & \\ \vdots & & & \vdots \\ c_n & & \cdots & c_0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

If T_N has rank N, then the problem of extending the matrix to a larger $n \times n$ matrix (which is equivalent to determining the higher Fourier coefficients of the spectrum f) is related to the problem of enlarging T_N to an $n \times n$ matrix T_n in a way than minimizes the *rank* of T_n.

Currently we are working on the problem of extending a band matrix (not necessarily Toeplitz or even selfadjoint) in a way that minimizes the rank of the extension. It appears that the minimum rank criterion is dual in some sense to the maximum entropy criterion.

We are also working on infinite-dimensional problems along the lines

described in the research proposal.

ACTIVITIES OF THE INVESTIGATORS, 1987-1988

During the Fall semester of 1987 we conducted a working seminar on operator theory. The material in the seminar focused on problems connected with our proposal. Many lectures were presented by Israel Gohberg and Joseph Ball, both of whom spent a sabbatical semester visiting the University of Maryland. The average attendance was 20, with nearly equal number of faculty members and graduate students.

Israel Gohberg is now in the process of writing with Joseph Ball and Leiba Rodman a book on interpolation problems for rational matrix functions.

During the Spring semester of 1988, Robert Ellis and David Lay were on sabbatical leave. Ellis spent time in Amsterdam and Tel Aviv, working with Gohberg, and gave lectures on our work at six universities in The Netherlands, Germany, Greece and Israel. Lay attended a course in signal processing, gave several lectures on our work and attended the SIAM Conference on Applied Linear Algebra in Madison, Wisconsin, and a four-day conference on Matrix Spectral Inequalities at Johns Hopkins University.

For ten weeks, beginning the first of June, 1988, we are conducting a working seminar on rational matrix functions and connections with the theory of linear systems. The seminar meets 4 to 5 hours per week. During this period several research associates will be participating, in addition to Ellis, Lay, and Gohberg:

Prof. Harm Bart - 2 months
    Erasmus University, Rotterdam
Prof. Reinhard Mennicken - 1 day
    University of Regensburg, West Germany
Prof. Boris Reichstein - 1 week
    Catholic University, Washington, D.C.
Prof. Rien Kaashoek - 3 weeks
    Free University, Amsterdam

Reports of our research at conferences has stimulated considerable interest in the problems we have described, and several other researchers are investigating related topics.

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